Understanding Sediment Transport Dynamics: Moving Towards a New Approach

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Abstract: During the last century, a great deal of research has been devoted to the development of approaches for modeling the dynamics of sediment transport phenomenon. In spite of the progress achieved, our understanding of the sediment transport phenomenon is far from complete, as there is no generally accepted relationship between the components involved, e.g. water discharge, suspended sediment concentration and bed load. Also, any error (e.g. measurement error) in one component of the system (e.g. discharge) could eventually lead to an inaccurate outcome in deriving either another component or relationships between the components. One possible way to avoid these problems is by modeling the component of interest (e.g. bed load) using a time series of the same component itself. Such an approach may also be able to represent the dynamics of the entire system. In this regard, the concept of phase-space reconstruction, i.e. reconstruction of a single-variable time series in a multi-dimensional phase-space to represent the underlying dynamics, and the related ideas of deterministic chaos theory could be useful. This study investigates the possible use of such an approach for understanding bed load dynamics. The approach is employed for predicting the bed load dynamics in the Mississippi River basin in USA. The predictions are made using a local approximation method. The predicted bed loads are found to be in very good agreement with the observed ones. The near-accurate predictions indicate the appropriateness of phase-space reconstruction and local approximation prediction for understanding the bed load dynamics. The results also reveal that the bed load dynamics are dominantly influenced by three variables, suggesting that the dynamics could be understood from a low dimensional chaotic dynamical perspective.

Keywords: Sediment transport; Modeling and prediction; Bed load; Phase-space; Local approximation prediction

1. INTRODUCTION

The twentieth century witnessed a large number of studies on the development of approaches for modeling and predicting the dynamics of sediment transport phenomenon. Most of these approaches revolve around linking the components involved in the underlying river system, e.g. water discharge, suspended sediment concentration and bed load [e.g. Lewis, 1921; Einstein, 1943; Milliman and Meade, 1983; Olive et al., 1996; Bull, 1997]. Notwithstanding the progress achieved using such approaches, our understanding of the sediment transport phenomenon is far from complete, essentially because there is no generally accepted relationship between the components. For example, studies report a variety of relationships, depending upon the characteristics of the river or water body, as follows: (a) peaks in suspended sediment concentration and discharge may coincide [e.g. Lewis, 1921]; (b) sediment peak lags discharge peak [e.g. Einstein, 1943]; and (c) sediment peak arrives before the discharge peak [e.g. Olive et al., 1996].

In view of the above, it appears necessary to devise a new approach to improve our understanding of the sediment transport phenomenon. An important requirement of such an approach is that it should be able to represent not only the dynamics of the individual components but also the relationships between them as well as their individual and combined influence on the sediment transport phenomenon. In this regard, the notion of deterministic chaos (i.e. seemingly complex irregular behavior could be the result of simple deterministic systems influenced by a few nonlinear interdependent variables sensitive to initial conditions) that employs the concept of phase-space reconstruction (i.e. reconstruction of a single-dimensional or variable time series in a multi-dimensional phase-space to represent the underlying dynamics) may be useful.

The ideas gained from the theory of chaos seem to be useful and appropriate for understanding the sediment transport phenomenon for the following reasons: (1) almost all of the components involved in the sediment transport phenomenon present some degree of nonlinearity; and (2) any small error (e.g. measurement error) in any one of the components (e.g. discharge) could eventually lead to a large error in the outcome (e.g. relationships between the components). The fact that applications of such ideas have yielded encouraging modeling and prediction results for a variety of hydrological phenomena, such as rainfall [e.g. Rodriguez-Iturbe et al., 1989; Jayawardena and Lai, 1994; Sivakumar et al., 1999 and 2001], runoff or discharge [e.g. Jayawardena and Lai, 1994; Porporato and Ridolfi, 1997], rainfall-runoff [e.g. Sivakumar et al., 2000], among others, is an additional driving force for their possible use in sediment transport phenomenon. A detailed discussion on the validity of the past studies employing the concept of chaos to hydrological phenomena and the reported results is made in Sivakumar [2000].

In the present study, in order to investigate the possible use of chaos theory for sediment transport phenomenon, the individual dynamical behavior of only one component, i.e. bed load, is studied. Daily bed load data observed at the Mississippi River basin in USA are analyzed. A multi-dimensional phase-space is reconstructed first using the bed load series in order to represent the dynamics of the underlying system, and predictions are then made employing a local approximation method, with a local polynomial approach.

2. PHASE-SPACE RECONSTRUCTION AND LOCAL APPROXIMATION PREDICTION

Phase-space is a useful tool for characterizing dynamical systems (whether low dimensional or high dimensional). A dynamical system can be described by a phase-space diagram, which is essentially a coordinate system (or a graph), whose coordinates are all the variables that enter the mathematical formulation of the system (i.e. the variables necessary to completely describe the state of the system at any moment). The trajectories of the phase-space diagram describe the evolution of the system from some initial state, which is assumed to be known, and hence represent the history of the system. A point in the phase-space represents the state of the system at a given time. Phase-space is a powerful concept because with a model and a set of appropriate variables, dynamics can represent a real-world system as the geometry of a single moving point.

For a dynamical system with known partial differential equations (PDEs), the system can be studied by discretizing the PDEs, and the set of variables at all grid points constitutes a phase-space, which is an approximation to the original (infinite-dimensional) phase-space. For such a system, an additional difficulty is that the initial values of many of the variables may be unknown. However, a time series of a single variable of such a complex system may be available, and this allows the attractor (a geometric object which characterizes the long-term behavior of a system in the phase-space) of the system to be reconstructed. The physics behind such a reconstruction is that a nonlinear system is characterized by self-interaction, so that a time series of a single variable can carry the information about the dynamics of the entire multi-variable system.

Though there exists a variety of methods for phase-space reconstruction, the method of delays [e.g. Takens, 1981] is the most common in use. According to this method, using its past history and an appropriate delay time, a single-variable time series X_i, where $i = 1, 2, ..., N$, can be reconstructed in a multi-dimensional phase-space, given by:

$$Y_j = (X_j, X_{j+\tau}, X_{j+2\tau}, ..., X_{j+(m-1)\tau}) \qquad (1)$$

where $j = 1, 2, ..., N-(m-1)\tau$, m is the dimension of the vector Y_j, called as embedding dimension; and τ is a delay time [Packard *et al.*, 1980; Takens, 1981].

A correct phase-space reconstruction in an embedding dimension m facilitates one to interpret the underlying dynamics in the form of an m-dimensional map, f_T, according to:

$$Y_{j+T} = f_T(Y_j) \quad (2)$$

where Y_j and Y_{j+T} are vectors of dimension m, describing the state of the system at times j (e.g. current state) and $j+T$ (e.g. future state), respectively. The problem then is to find an appropriate expression for f_T (e.g. F_T).

The approaches that exist for determining F_T may broadly be divided into two categories, namely global approximation and local approximation. In the global approximation approach, the map (Eq. 2) is approximated by working globally on all over the attractor and seeking a map F_T that is valid at every point of it. On the other hand, local approximation [e.g. Farmer and Sidorowich, 1987] entails the subdivision of the f_T domain into many subsets (i.e. neighborhoods), each of which identifies some approximations F_T, valid only in that same subset. In other words, the dynamics of the system is described step by step locally in the phase-space. An important advantage of the local approximation approach over the global one is that it leads to a considerable reduction in the complexity of the representation F_T, without degrading the quality of the forecast, to the point that, for the very short term, it provides generally better results than those obtainable using the global approximation. The local approximation approach is employed in the present study for predicting the daily bed load in the Mississippi River basin. The prediction algorithm used in this study is as follows.

The identification of the sets in which to sub-divide the domain is done by fixing a metric || ||, and then, given the starting point Y_j from which the forecast is initiated, identifying neighbors Y_j^p, $p = 1, 2, ..., k$, with $j^p < j$, nearest to Y_j, which constitute the set corresponding to the point Y_j. The local functions can then be built, which take each point in the neighborhood to the next neighborhood: Y_j^p to Y_{j+1}^p. The local map F_T, which does this, is determined by a least squares fit minimizing

$$\sum_{p=1}^{k} \| Y_{j+1}^{p} - F_T Y_j^{p} \|^2 \quad (3)$$

The local maps are then learned in the form of local polynomials [e.g. Abarbanel, 1996], and predictions are made forward from a new point Z_0 using these local maps. For the new point Z_0, the nearest neighbor in the training set is found, which is denoted as Y_q. Then the evolution of Z_0 is found, which is denoted as Z_1 and is given by:

$$Z_1 = F_q(Z_0). \quad (4)$$

Then the nearest neighbor to Z_1 is found, and the procedure is repeated to predict the subsequent values. The algorithm is implemented herein using the cspW software [Randle Inc., 1996].

The accuracy of prediction is evaluated using three statistical indicators, namely correlation coefficient (CC), root mean square error ($RMSE$), and coefficient of efficiency (R^2). The time series plots and the scatter diagrams are also used to choose the best prediction results among a large combination of results achieved with different embedding dimension.

3. ANALYSES AND DISCUSSION OF RESULTS

3.1 Study Area and Data Used

The river basin considered in the present study is the Mississippi River basin in USA. The Mississippi River is a dominant mover of sediment and the River system transports more sediment than any other river in North America. In spite of the large dams that have been built across its major tributaries, the Mississippi River still ranks sixth in the world in suspended sediment discharge to the oceans [e.g. Milliman and Meade, 1983]. The average annual sediment discharge to the coastal zone by the Mississippi River is about 230 million tons.

Throughout the Mississippi River basin, discharge and sediment are measured at a number of locations. In the present study, bed load data observed at a gaging station at St. Louis in the State of Missouri (U.S. Geological Survey station no. 07010000) is studied. The basin is situated between 38°37′03″ latitude and 90°10′47″ longitude, on downstream side of west pier of Eads Bridge at St. Louis. The area of the basin is 251,230 km^2 [e.g. Chin et al., 1973].

Even though, for the above station, daily bed load measurements are available from April 1948, there were some missing data before 1960 and also after 1981. In order to avoid the uncertainties that could arise on the outcomes due to such missing data, the period of only continuous data, i.e. from January 1961 to December 1981, is considered.

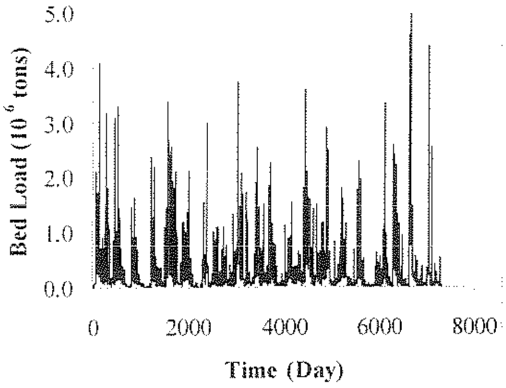


Figure 1. Variation of daily bed load data in the Mississippi River basin, USA.

The variation of the daily bed load series for a period of 20 days (from January 1961 to December 1980) is shown in Figure 1, and Table 1 presents some of the important statistics of the series.

Table 1. Statistics of daily bed load data from the Mississippi River basin, USA.

Statistic	Value (tons)
Mean	283205
Standard deviation	422233
Maximum value	4960000
Minimum value	2540

3.2 Analyses and Discussion of Results

Figure 2 presents the reconstruction of the single-variable bed load series in a two-dimensional phase-space (m = 2), i.e. the projection of the attractor on the plane $\{X_i, X_{i+1}\}$. As can be seen, the reconstruction yields a reasonably well-defined attractor, with the exception of the presence of only a few outliers corresponding to very high values in the series.

The local approximation method, with a local polynomial approach, is now employed for predicting the daily bed load series. The entire data set of 21 years is divided into two parts; the first 20 years (January 1961 – December 1980) of data are used in the phase-space reconstruction (i.e. training) and predictions are made for the subsequent 1-year (January 1981 – December 1981) of data. Embedding dimensions from 1 to 10 are used for the reconstruction purposes, and predictions are made for one-time step ahead.

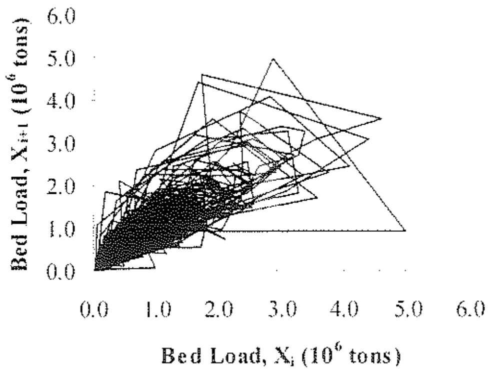


Figure 2. Phase-space diagram of daily bed load data in the Mississippi River basin, USA.

Table 2 and Figure 3 present a summary of the prediction results achieved for the bed load series, in terms of correlation coefficient (CC), root mean square error ($RMSE$), and coefficient of efficiency (R^2). As can be seen from the statistics in Table 2, overall reasonably good predictions (with $CC > 0.97$, $R^2 > 0.92$) are achieved for the bed load series, for all the ten embedding dimensions. A further insight at the statistics reveals that the better predictions are achieved when the embedding dimensions are less than or equal to four.

In regards to the selection of the optimal embedding dimension, $mopt$, a comparison, using time series plots and scatter diagrams, of the observed values with the predicted ones, obtained for the ten embedding dimensions, reveals that the best prediction results are indeed achieved when the embedding dimension is three, i.e. m_{opt} = 3 (with CC = 0.993, $RMSE$ = 62010 tons/day, R^2 = 0.979). These results seem to indicate that a 3-dimensional phase-space is reasonably sufficient to capture the important features of the bed load dynamics; in other words, the bed load dynamics may be dominantly dependent on only three variables (or components).

Table 2. Prediction results for daily bed load data from the Mississippi River basin, USA.

*m**	*CC*	*RMSE*	R^2
1	0.992	56957	0.982
2	0.990	66484	0.976
3	0.993	62010	0.979
4	0.994	63341	0.978
5	0.991	77289	0.967
6	0.984	91645	0.954
7	0.982	97518	0.948
8	0.981	104222	0.941
9	0.979	110121	0.934
10	0.976	114084	0.929

*m = Embedding Dimension; CC = Correlation coefficient; RMSE = Root Mean Square Error; and R2 = Coefficient of Efficiency

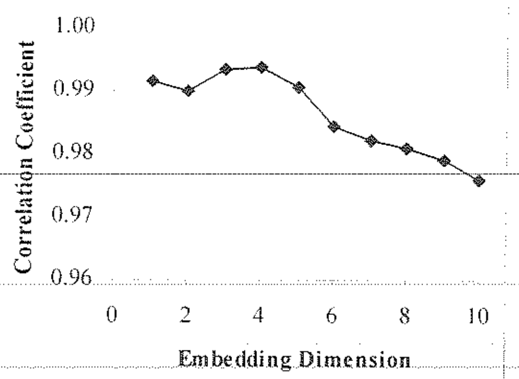
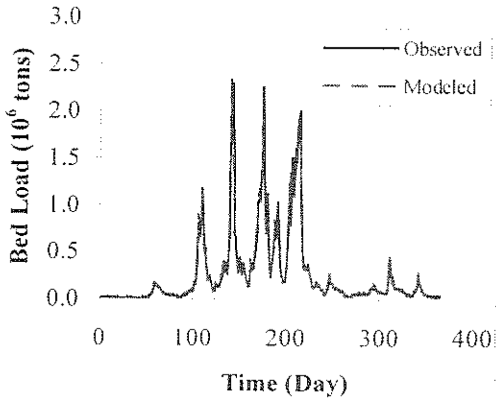


Figure 3. Prediction accuracy versus embedding dimension for daily bed load data in the Mississippi River basin, USA.

To illustrate further on the effectiveness of the local approximation prediction method, Figure 4(a), for instance, presents a comparison, using time series plot, of the predicted and the observed bed load values. The plot shown corresponds to the predictions obtained when the time series is reconstructed in a three-dimensional phase space. As can be seen, the predicted values are, in general, in good agreement with the observed ones. A closer look at the two (i.e. observed and predicted) time series reveals that the local polynomial prediction approach captures both the major trends as well as the minor (noisy) fluctuations in the bed load series. The ability of the local approximation procedure in the prediction of bed load dynamics lies essentially in representing the dynamics captured in the phase-space step by step in local neighborhoods. The good agreement between the observed and predicted series can also be revealed by plotting the scatter diagrams, shown in Figure 4(b) for the above case (i.e. embedding dimensions = 3), where the solid 1:1 (diagonal) line plotted for reference.



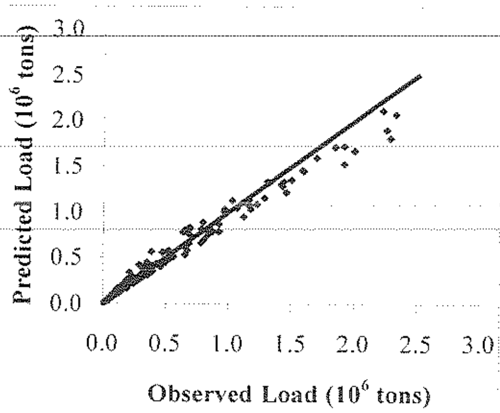


Figure 4. Comparison of observed and predicted daily bed load in the Mississippi River basin, USA: (a) Time series plot; and (b) Scatter diagram. Embedding dimension (*m*) = 3.

4. SUMMARY AND CONCLUSIONS

An attempt was made to investigate the possible use of the concepts of phase-space reconstruction and deterministic chaos theory for understanding and predicting the dynamics of sediment transport phenomenon. Only one component of the sediment transport system, namely bed load, was studied. By reconstructing the scalar bed load series in a multi-dimensional phase-space the dynamics of the underlying system was represented first, and then a local approximation method (with a local polynomial approach) was employed for making predictions of bed load. Daily bed load data observed in the Mississippi River basin in USA were analyzed.

The phase-space reconstruction and local approximation prediction yielded near-accurate predictions of the bed load dynamics (with $CC > 0.99$ and $R^2 > 0.97$). They captured well not only the major trends in the bed load dynamics but also the minor fluctuations and the extreme values. The prediction results also revealed that the reconstruction of the scalar bed load series in a 3-dimensional phase-space would be reasonably sufficient to capture the important features of the bed load dynamics. A possible implication of such results is that the dynamics of bed load (and other sediment transport related) phenomena could be viewed from a low dimensional (chaotic) dynamical perspective. The immediate task, in the wake of these encouraging results, is to study the dynamics of other components of the sediment transport system as well in order to provide further evidence and support the present results. If successful, the ideas may also be employed for a variety of river systems in different climatological and geographical regions.

5. REFERENCES

Abarbanel, H.D.I., *Analysis of Observed Chaotic Data*, Springer-Verlag, New York, USA, 1996.

Bull, L.J., Relative velocities of discharge and sediment waves for the River Severn, UK, *Hydrological Sciences Journal*, 42(5), 649-660, 1997.

Chin, E.H., J. Skelton, and H.P. Guy, The 1973 Mississippi River Basin flood: Compilation and analysis of meteorologic, streamflow, and sediment data, U.S. Geological Survey Professional Paper, 937, pp. 1-137, 1973.

Einstein, H.A., Formulas for the transportation of bedload, *Journal of Hydraulic Division, ASCE*, 107, 561-597, 1943.

Farmer, D.J., and J.J. Sidorowich, Predicting chaotic time series, *Physical Review Letters*, 59, 845-848, 1987.

Jayawardena, A.W., and F. Lai, Analysis and prediction of chaos in rainfall and stream flow time series, *Journal of Hydrology*, 153, 23-52, 1994.

Lewis, A.D., Silt observations of the River Tigris, *Institution of Civil Engineers, Minutes of the Proceedings*, 212, 393-399, 1921.

Milliman, J.D., and R.H. Meade, Worldwide delivery of river sediments to the oceans, *Journal of Geology*, 91, 1-9, 1983.

Olive, L.J., J.M. Olley, P.J. Wallbrink, and A.S. Murray, Downstream patterns of sediment transport during floods in the Murrumbidgee River, NSW, Australia, *Z. für Geomorphologie, Supplementband*, 105, 129-140, 1996.

Packard, N.H., J.P. Crutchfield, J.D. Farmer, and R.S. Shaw, Geometry from a time series, *Physical Review Letters*, 45(9), 712-716, 1980.

Porporato, A., and L. Ridolfi, Nonlinear analysis of river flow time sequences, *Water Resources Research*, 33(6), 1353-1367, 1997.

Randle Inc., *cspW, Tools for Dynamics*, Applied Nonlinear Sciences, Randle Inc., Great Falls, Virginia, USA, 1996.

Rodriguez-Iturbe, I., F.B. De Power, M.B. Sharifi, and K.P. Georgakakos, Chaos in rainfall, *Water Resources Research*, 25(7), 1667-1675, 1989.

Sivakumar, B., Chaos theory in hydrology: important issues and interpretations, *Journal of Hydrology*, 227(1-4), 1-20, 2000.

Sivakumar, B., S.Y. Liong, C.Y. Liaw, and K.K. Phoon, Singapore rainfall behavior: chaotic? *Journal of Hydrologic Engineering, ASCE*, 4(1), 38-48, 1999.

Sivakumar, B., R. Berndtsson, J. Olsson, K. Jinno, and A. Kawamura,, Dynamics of monthly rainfall-runoff process at the Göta basin: A search for chaos, *Hydrology and Earth System Sciences*, 4(3), 407-417, 2000.

Sivakumar, B., S. Sorooshian, H.V. Gupta, and X. Gao, A chaotic approach to rainfall disaggregation, *Water Resources Research*, 37(1), 61-72, 2001.

Takens, F., Detecting strange attractors in turbulence, In: D.A. Rand and L.S. Young (eds), Dynamical Systems and Turbulence, *Lecture Notes in Mathematics 898*, Springer-Verlag, Berlin, pp. 366-381, 1981.